Sustainable Debt Restructuring with Investment and Non-Elite Participation

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Abstract

Should the consequences of the Covid 19 shock on developing countries be treated as a temporary shock or as one with potentially longer-term consequences and if so, what are the implications for debt restructuring? Is new financing required to ensure sustainable debt restructuring? How is sustainable debt restructuring impacted by creditor heterogeneity? How should issues of debtor moral hazard be addressed? In this paper, we provide a theoretical analysis of these issues. Our broad conclusions are as follows: (a) sustainable debt restructuring must involve a mix of debt write down and financing in the form of outright grants and loans at very low interest rates, and (b) participation in the debt restructuring process by community groups, civil society organisations is key to restoring sustainability.

Keywords: Debt, restructuring, sustainability, investment, negative shock, elite, non-elite, UNCTAD road map.

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1. Introduction

Even before the global negative shock resulting from the Covid 19 pandemic, in a recent report the World Bank (2020) reports that total developing country in 2018, an increase of 54 percentage points of GDP since 2010. In the first wave of the pandemic, the COVID-19 crisis led to a sudden collapse in capital flows to emerging and developing countries. The Institute of International Finance estimates that portfolio outflows from emerging market countries amounted to nearly $100 billion over a period of 45 days starting in late February 2020 (IIF 2020). Moreover, the countries at high or moderate risk of debt distress are disproportionately fragile and conflict-affected states, commodity-dependent countries, and small States. None of this takes account the potential negative consequences of a second wave of the pandemic unfolding globally.

This state of affairs has led to calls for immediate payment standstills (with debt payments postponed in the short-term (see, for example, Bolton et.al. (2020)). As the World Bank Report referred to before points out, "Debt service suspension is a powerful, fast-acting measure that can bring real benefits to people in poor countries, particularly countries that don't have the financial resources to respond to the coronavirus (COVID-19) crisis." The IMF and the World Bank have been proactive in implementing a debt standstill. The G20 has agreed to a debt service standstill on bilateral loans for a group of 76 low-income countries. Some private debt has also been rolled over (Eichengreen 2020).

However, should the consequences of the Covid 19 shock on low income countries be treated as a temporary shock or as one with potentially longer-term consequences and if so, what are the implications for debt restructuring? How is sustainable debt restructuring impacted by creditor heterogeneity? And, is new net financing required to ensure sustainable debt restructuring? How should issues of debtor moral hazard be addressed?

In this paper, our aim is to provide a formal analysis of sustainable debt restructuring in the face an unanticipated negative shock such as Covid 19 where the medium-term consequences of the unanticipated negative shock depend on the debt restructuring proposal specifically the financing proposed as part of the debt restructuring plan.

We begin by examining the sustainability of a debt restructuring proposal where the interest rate at which the debt is restructured is not contingent on recovery.[2] We show that that a necessary and sufficient condition for a non-contingent debt restructuring proposal to be sustainable is that the interest rate at which the debt is restructured is lower than the expected rate of growth by a factor that takes into account the existing stock of debt. We show that when investment has a positive impact on growth, at a given level of financing, there is an upper bound on the interest rate at which debt can be sustainably restructured.

We, then, use our results to examine the sustainability of a debt standstill (with partial payments standstill) and its alternatives. Using data from a recent report by the United Nations Economic Commission for Africa (UNECA (2020)), we

[2] In Appendix 2, we examine the sustainability of contingent debt restructuring. We show that for any level of financing, there is always a sustainable contingent debt restructuring proposal with positive expected present value. However, we also show that there cannot be a sustainable contingent debt restructuring proposal with the same expected present value as an unsustainable non-contingent debt restructuring proposal.

carry out a simple calibration to quantify the impact of changes in the level of financing on the upper bound on the interest rate at which debt can be sustainably restructured. Our calibration exercise shows that under plausible assumptions on the existing stock of debt and the financing needs, there is no positive interest rate at which sustainable debt restructuring can take place. Debt restructuring must involve a mix of debt write down and financing in the form of outright grants and loans at very low interest rates.

We address the issue creditor heterogeneity and participation. When creditors do not act collectively and, for example, disbursements made by official creditors are used, by the debtor state, to make payments to private creditors, we show that sustainability conditions must become more restrictive and use our calibration exercise to quantify the effects.

We consider the issue of debtor moral hazard from the perspective of its domestic political economy. Domestic agents are split into two groups, a minority elite and a majority non-elite. Elites are initially organised to act collectively in their own interest and have access to international capital markets (so that they can divert a portion of the financing via debt restructuring for their own private consumption); non-elites aren't initially organised collectively and cannot access international financial markets: hence, if they have decision-making power they would use the financing via debt restructuring for poverty reduction, public health and growth enhancing projects. We characterize the conditions under which the non-elite are able to organise effectively to acquire decision-making power within the debtor state. Our formal analysis implies that the participation in the debt restructuring process by citizens of the debtor state is key to restoring sustainability. This leads us to suggest an UNCTAD road map (UNCTAD 2015) for Africa as a key component of debt restructuring in response to Covid 19 where women, public health practitioners, key workers, community groups, civil society organisations participate in the debt restructuring process to determine the financing needs of the debtor state, the interest rate at which the financing occurs as well as the use of the finance within the debtor state.

The remainder of the paper is structured as follows. Section 2 presents an analysis of sustainable debt restructuring. Section 3 uses the analysis of Section 2 to examine the sustainability of a debt standstill and its alternatives such as a debt write down both analytically and via a calibration; it contains an analysis of creditor participation. Section 4 is devoted to an analysis of the political economy of debtor moral hazard and, in the light of this analysis, examines the role that could be played by the UNCTAD road map to resolve issues of debtor moral hazard. The last section concludes. Appendix 1 shows how the two-period model used in the main text can be derived from an endogenous growth model. Appendix 2 contains analysis of contingent debt.

2. Characterizing Sustainable Debt Restructuring in a simple model

Take the case of a country embarking on a bond-financed investment project, costing[3] K, which lasts only two periods $t = 1,2$. All the finance is supplied by external creditors who are promised returns of $(1 + r)$ in the first period and $(1 + r)^2$ in

[3] In what follows, the unit of measurement is USD at current exchange rates or some such other basket of international currencies consistent with the point that the finance is supplied by external creditors: the sovereign is unable to issue debt in its own currency. All expressions are per capita given the population of the debtor state.

the second period. So long as resources are available cover these payments (i.e. cash flow in period 1 is greater than $(1+r)K$ and cash flow in period 2 is greater than $(1+r)^2K$, all is well and the project will run to completion.

Consider what happens if an unanticipated, exogenous shock (e.g. the Covid 19 pandemic 'bad luck') lowers the capacity to pay in period 1 below the amount that is due to bond holders i.e. $y_1 - \underline{y} < (1+r)K$ where y_1 denotes the resources available to make debt payments (e.g. tax revenues or export earnings) in period 1 and $\underline{y}$ is the minimum consumption of these resources for domestic purposes by the debtor state in period 1.

Conditional on the negative shock, there is uncertainty about whether project net worth will be unchanged. This will depend on beliefs about the prospect of recovery i.e. whether the negative shock will turn out to be temporary or permanent. A temporary shock would correspond to the notion that economies, globally, will emerge, more or less simultaneously from the lockdown due to Covid 19 in a V-shaped recovery. An alternative view is that as economies adjust to the shock, there will be permanent changes to the global economy due to changes in global supply chains, permanent damage to the health of its populations making it a less attractive destination for foreign direct investment, or tourism etc.

Assuming that $0 \leq I \leq (1+r)K - y_1 + \underline{y} + X$ is advanced to the sovereign where$(1+r)K - y_1 + \underline{y}$ is the amount owed (a fraction of which) is rolled over and $X \geq 0$ is new financing, the probability of recovery is denoted by $p(I)$ (with probability $1 - p(I)$ the shock has permanent negative consequences): recovery implies that at $t = 2$, the total capacity to pay is $y_H - \underline{y}$ (without recovery, available resources are $y_L - \underline{y}$ with $y_L < y_H$). Throughout the paper, we will assume that $p(I)$ is an increasing concave function of investment with $p(0) = 0, p'(I) > 0, p''(I) < 0$. The idea is that the resources freed up by debt restructuring and new financing will be used by the sovereign state to store up its capacity to generate resources (e.g. by investing in public health) as it emerges from Covid 19.[4]

Formally, we define a debt restructuring proposal as follows:

Definition 1. The pair (r', I) is a debt restructuring proposal.

Given (r', I), the present value of the refinanced debt will be $(1+r')I + (1+r')^2K$. In expected terms, the total resources available is $p(I)\left[y_H - \underline{y}\right] + (1 - p(I))\left[y_L - \underline{y}\right]$. Hence, for restructuring to be sustainable (in expected terms), we must have

$$(1+r')I + (1+r')^2K \leq p(I)\left[y_H - \underline{y}\right] + (1 - p(I))\left[y_L - \underline{y}\right] \ldots(1)$$

In (1) the assumption is that all money advanced to the sovereign state is used for growth enhancing investment by the sovereign state. In practice, of course, this assumption is likely to be violated for two reasons. First, the money advanced to the sovereign could be used to bail out individual creditors: so, money advanced by the IMF could be used to bail out

[4]We work in a two period setting in the main text. In Appendix 1, we show how the our two period analysis can be derived from a simple AK endogenous growth model.

private creditors. Second, there is the issue of the debtor moral hazard as the some or all the money could be used for non-productive investment or private consumption. For the moment, we abstract from both these activities; however, we address these two issues later in the paper (section 3.4 for creditor incentives and sections 4.1 and 4.2 for the political economy of debtor moral hazard).

As $0 \leq p(I) \leq 1$, when a debt restructuring proposal is sustainable, at a minimum, the rolled over payment has to be feasible when recovery takes place i.e. $(1+r')I + (1+r')^2 K \leq y_H - \underline{y}$; however, as (1) holds in expected terms only, without recovery, creditors as a group receive $min\left\{y_L - \underline{y}, (1+r')I + (1+r')^2 K\right\}$.

Given a debt restructuring proposal (r', I), the expected return is $\pi(I) = p(I)\left[y_H - \underline{y}\right] + (1-p(I))\left[y_L - \underline{y}\right]$. Expected growth is given by the expression:

$$EG(I) = \frac{p(I)\left[y_H - \underline{y}\right] + \left(1-p(I)\right)\left[y_L - \underline{y}\right]}{I} = \frac{\pi(I)}{I}$$

and the expected rate of growth is $Eg(I) = EG(I) - 1$.

Next we define an admissible debt restructuring proposal:

Definition 2. The pair (r', I) is an admissible debt restructuring proposal if $(1+r') \geq 0$ i.e. the present value of restructured debt is non-negative.

The following proposition provides a basic characterization for an admissible, sustainable debt restructuring proposal (r', I):

Proposition 1: *A necessary and sufficient condition for an admissible debt restructuring proposal (r', I) to be sustainable is $1 + r' \leq 1 + Eg(I) - \varepsilon(I)$ where $0 < \varepsilon(I) < 1 + Eg(I)$.*

Proof. Let $x = (1+r')$. Note that we can rewrite (1) as

$$x + x^2 \frac{K}{I} \leq EG(I)$$

As $x^2 \frac{K}{I} \geq 0$, we have that $x = (1+r') \leq EG(I) = (1 + Eg(I)$ so that $r' \leq Eg(I)$. Next, let $\varepsilon(I) = EG(I) - x$. Instead of computing the roots of the equation

$$x + x^2 \frac{K}{I} = EG(I)$$

We, instead, compute the roots of the equation

$\left(EG(I) - \varepsilon(I)\right) + \left(EG(I) - \varepsilon(I)\right)^2 \frac{K}{I} - EG(I) = 0$ ……..(2)

By computation, note that

$$\left(EG(I)-\varepsilon(I)\right)+\left(EG(I)-\varepsilon(I)\right)^2\frac{K}{I}-EG(I)=0\leftrightarrow-\varepsilon(I)+\left(EG(I)-\varepsilon(I)\right)^2\frac{K}{I}=0$$

$$\leftrightarrow\left(EG(I)\right)^2+\left(\varepsilon(I)\right)^2-\left(2EG(I)+\frac{I}{K}\right)\varepsilon(I)=0$$

Hence, the roots of equation (2) are given by expression

$$\varepsilon(I)=\frac{\left(2EG(I)+\frac{I}{K}\right)\pm\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}$$

As

$$\left(2EG(I)+\frac{I}{K}\right)^2=\left(\frac{I}{K}\right)^2+4\left(EG(I)\right)^2+\frac{4EG(I)I}{K}>\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}$$

it follows that

$$\left(2EG(I)+\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}>0$$

Hence, both roots are positive. Consider, the first root:

$$\varepsilon(I)=\frac{\left(2EG(I)+\frac{I}{K}\right)+\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}$$

By computation, we obtain that:

$$EG(I)-\varepsilon(I)=\frac{-\left(\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}<0$$

This implies we must have $(1+r')=EG(I)-\varepsilon(I)<0$ so that the restructured has negative present value and therefore, cannot be admissible. Consider the second root:

$$\varepsilon(I)=\frac{\left(2EG(I)+\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}$$

By computation:

$$EG(I)-\varepsilon(I)=\frac{-\left(\frac{I}{K}\right)+\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}>0$$

as $\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}>\left(\frac{I}{K}\right)$. Hence, if we take the second root, we have that the present value of the restructured debt is non-negative and when (1) holds, as the LHS of (1) is increasing in r',

$$1+r'\leq EG(I)-\varepsilon(I)=1+Eg(I)-\varepsilon(I)$$

as required. ■

The above proposition provides a necessary and sufficient condition for debt restructuring to be sustainable. The intuition is as follows. Given that I is advanced to the debtor state for investment at interest rate r', for debt to be sustainable, the expected growth rate must be higher than r' by a factor that takes into account the fact that $(1 + r')K$ must still be repaid. Where the interest rate and the growth rate are measured in a common unit of account, in general multi-period models of debt sustainability (see, for example, Contessi (2012)) a similar condition is imposed to ensure that the time path of debt remains bounded i.e. debt does not explode. However, the point is that the upper bound on the interest rate at which debt is restructured may imply that $1 + r' < 1$ so that it involves a mix of debt write down and additional finance in the form of outright grants and loans at very low interest rates such as disbursements under the Rapid Credit Facility (RCF) at zero interest rates.

The following diagram illustrates this condition characterizing sustainable debt restructuring:

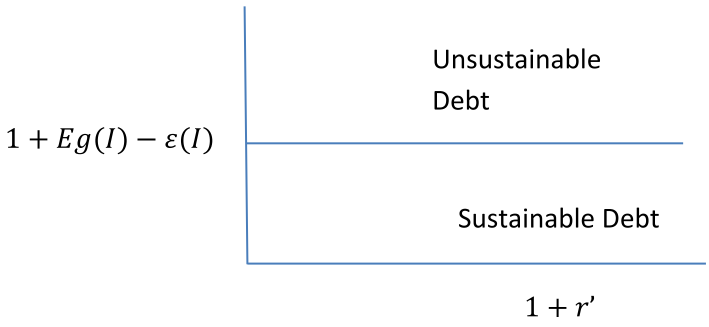

Figure 1: Sustainable Debt Restructuring

Devdas and Pennings (2018) provide evidence that the return to new public investment can be as much as 9% in low income countries even accounting for inefficiency and depreciation. Furthermore, there is considerable evidence that growth is itself increasing in public investment in low income countries as much as is middle income and high income countries although exact estimates vary and depend on the estimation method as well as the time horizon (see, for example, Pereira and Andraz (2013), Devadas and Pennings (2018)): the longer the time horizon, the higher the effect of an increase in public investment on growth.

Under the assumption that expected growth is increasing in investment, the next proposition provides an additional, useful characterization of sustainable admissible debt restructuring proposals where either the interest rate or the amount of investment or both is taken as given:

Proposition 2: *Suppose expected growth is increasing in investment i.e.* $EG'(I) > 0$. *Then,* $EG'(I) - \varepsilon'(I) > 0$ *and for each* I *(respectively,* r' *s.t.* $1 + r' > 0$*), there exists an upper bound* $\hat{R}(I)$ *(resp.,* $\hat{I}(r')$*) such that* (r', I) *is an admissible sustainable debt restructuring proposal if and only if* $1 + r' \leq \hat{R}(I)$ *with* $\hat{R}'(I) > 0$ *(resp.,* $I \geq \hat{I}(1 + r')$ *with* $\hat{I}'(1 + r') > 0$*) and moreover,* $\hat{R}(I) = \hat{I}(\hat{R}(I))$ *and* $\hat{R}\left(\hat{I}(1 + r')\right) = \hat{I}(1 + r')$.

Proof. By Proposition 1, we need to check that $EG'(I) - \varepsilon'(I) > 0$. By computation:

$$EG'(I) - \varepsilon'(I) = \frac{1}{2}\left\{-\frac{1}{K} + \frac{1}{2}\left[\left(\left(\frac{I}{K}\right)^2 + \frac{4EG(I)I}{K}\right)^{-\frac{1}{2}}\right]\left[\frac{2I}{K^2} + \left(\frac{4}{K}\left(EG'(I)I + EG(I)\right)\right)\right]\right\}$$

Hence, we check that the expression within the curly brackets is greater than zero or equivalently

$$\frac{1}{2}\left[\left(\left(\frac{I}{K}\right)^2 + \frac{4EG(I)I}{K}\right)^{-\frac{1}{2}}\right]\left[\frac{2I}{K^2} + \left(\frac{4}{K}\left(EG'(I)I + EG(I)\right)\right)\right] > \frac{1}{K}$$

We can rewrite the above inequality as

$$\frac{K}{2}\left[\frac{2I}{K^2} + \left(\frac{4}{K}\left(EG'(I)I + EG(I)\right)\right)\right] > \left(\left(\frac{I}{K}\right)^2 + \frac{4EG(I)I}{K}\right)^{\frac{1}{2}}$$

Or equivalently as

$$\left[\frac{I}{K} + \left(2\left(EG'(I)I + EG(I)\right)\right)\right] > \left(\left(\frac{I}{K}\right)^2 + \frac{4EG(I)I}{K}\right)^{\frac{1}{2}}$$

Which, by taking the square on both sides, reduces to

$$\left(\frac{I}{K}\right)^2 + 4(EG'(I)I)^2 + 4\left(EG(I)\right)^2 + 4EG'(I)IEG(I) + \frac{4EG'(I)I^2}{K} + \frac{4EG(I)I}{K} > \left(\frac{I}{K}\right)^2 + \frac{4EG(I)I}{K}$$

Evidently, we can cancel $\left(\frac{I}{K}\right)^2$ and $\frac{4EG(I)I}{K}$ from both sides of the inequality. Therefore as long as $EG'(I) \geq 0$ and we must have that

$$4(EG'(I)I)^2 + 4\left(EG(I)\right)^2 + 4EG'(I)IEG(I) + \frac{4EG'(I)I^2}{K} + \frac{4EG(I)I}{K} > 0$$

which implies $EG'(I) - \varepsilon'(I) > 0$ as required. As $EG'(I) - \varepsilon'(I) > 0$, the admissible root to the equation Now, consider the admissible root of the equation $x + x^2\frac{K}{I} = EG(I)$ which, by Proposition 1, is $EG(I) - \varepsilon(I)$. As $EG'(I) - \varepsilon'(I) > 0$, it follows that (a) for a given value of I there is a unique solution $\hat{R}(I)$ to the preceding equation which is increasing in I, and (b) for a given value of $x > 0$, there is a unique solution $\hat{I}(x)$ to the preceding equation which is increasing in x. Moreover, by construction, $\hat{R}(I) = \hat{I}(\hat{R}(I))$ and $\hat{R}\left(\hat{I}(1 + r')\right) = \hat{I}(1 + r')$. ■

The intuition behind the preceding proposition is straightforward. Given that public investment has a positive impact on growth, given a level of investment, it follows that (a) there is an upper bound on the interest rate at which debt can be sustainably restructured, and vice versa (b) given the interest rate at which debt is restructured, there is a lower bound on the level of investment required to ensure sustainability. So again we can depict the characterization of a sustainable debt restructuring proposal in a diagram as follows[5]:

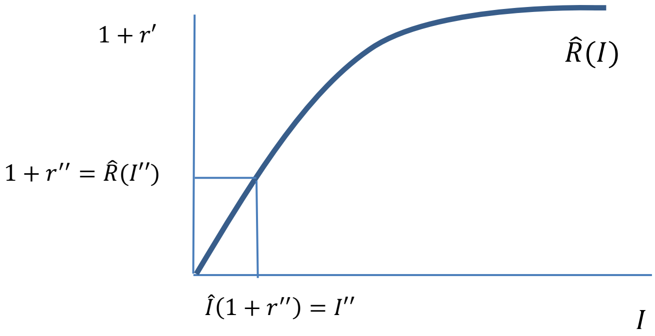

Figure 2. Upper bounds on the interest rate and lower bounds on investment

The above diagram illustrates the lower bound on investment given an interest rate r'' and vice versa, the upper bound on interest rate given the investment I'' consistent with Proposition 2. Of course, as $I'' = \hat{I}(1 + r'')$ it must follow that $1 + r'' = \hat{R}(I'')$.

In the next section, we use these two core results to evaluate a number of proposals that have been made in relation to debt restructuring in the face of Covid 19.

3. Evaluating the Sustainability of a Debt Standstill and its Alternatives

A key issue in current discussions around sovereign debt restructuring in the face of Covid 19 is whether a debt restructuring proposal should treat the shock as temporary or as one with potentially permanent consequences. We begin by using our results to evaluate the sustainability of a debt standstill; specifically, we characterize the conditions under which debt sustainability is restored with a debt standstill and examine whether restoring debt sustainability requires a debt write down or new investment both (Section 3.1), carry out a calibration exercise (Section 3.2) and examine issues of creditor participation (Section 3.3).

3.1 Debt Standstill, Debt Write-down and New Investment

Several commentators have called for an immediate payments standstill (with debt payments postponed in the short-term (see, for example, Bolton et.al. (2020)). On April 15, the G20 announced the Debt Service Suspension Initiative (DSSI), which allows the world's poorest countries — most of them in Africa — to suspend up to $14 billion of debt service payments due in 2020 for a group of 76 low-income countries. Some private debt has also been rolled over (Eichengreen 2020).

We examine the conditions under which a debt standstill will lead to sustainable debt restructuring. In our model, with our focus on debt sustainability, the permanent consequences of the shock are endogenous i.e. it depends on the actual debt

[5] The assumption of the concave shape of the function $\hat{R}(I)$ is a matter of convenience; in general, it can be either concave or convex or neither.

restructuring plan being proposed or implemented. In a debt standstill, all or some repayments on existing debt will be temporarily suspended and rolled over to the following period at the original interest rate. Formally:

Definition 3. A debt standstill with all or some payments suspended is a debt restructuring proposal (r', I) with $r' = r$ and $I \in [0, (1+r)K - y_1 + \underline{y}]$.

For such a debt standstill to be sustainable (in expected terms) the following inequality must hold for some $I \in [0, (1+r)K - y_1 + \underline{y}]$:

$$(1+r)I + (1+r)^2K \leq p(I)\left[y_H - \underline{y}\right] + \left(1 - p(I)\right)\left[y_L - \underline{y}\right] \text{...(3)}$$

The following proposition characterizes the conditions under which a debt standstill *isn't* sustainable:

Proposition 3: *For $I = (1+r)K - y_1 + \underline{y}$, if $1 + r > 1 + Eg(I) - \varepsilon(I)$ (equivalently, $1 + r > \hat{R}(I)$ when $G'(I) > 0$), then a debt standstill cannot be sustainable.*

Proof. By Proposition 1, note that a necessary and sufficient condition for a debt standstill to be sustainable is $1 + r \leq 1 + Eg(I) - \varepsilon(I)$ for some $I \in [0, (1+r)K - y_1 + \underline{y}]$. By Proposition 2, when $G'(I) > 0$, $1 + r > 1 + Eg(I)$, $I = (1+r)K - y_1 + \underline{y}$, it must be the case that $1 + r > 1 + Eg(I)$ for all $I \in [0, (1+r)K - y_1 + \underline{y}]$. Hence, a debt standstill cannot be sustainable when $1 + r > 1 + Eg(I) - \varepsilon(I)$ for $I = (1+r)K - y_1 + \underline{y}$. By Proposition 2, when $G'(I) > 0$ for $I = (1+r)K - y_1 + \underline{y}$, a debt standstill (r', I) is sustainable if and only if $1 + r \leq \hat{R}(I)$. The conclusion follows. ■

Proposition 3 is a straightforward consequence of Propositions 1 and 2. Its underlying intuition is best illustrated using the following diagram (where, for simplicity, the assumption is that entire amount owed in period 1 is rolled over):

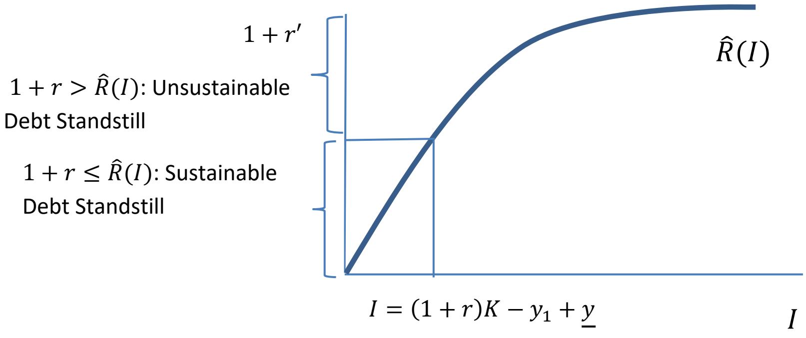

Figure 3. Sustainable and Unsustainable Debt Standstills

When a debt standstill isn't sustainable, restoring sustainability requires either $r' < r$ or $I > (1+r)K - y_1 + \underline{y}$ or both.

When $r' < r$ in a debt standstill, the present value of the restructured debt must be lower than the value of the original debt i.e.

$$(1+r')\left[(1+r)K - y_1 + \underline{y}\right] + (1+r')^2 K < (1+r)\left[(1+r)K - y_1 + \underline{y}\right] + (1+r)^2 K$$

which, of course, entails a haircut.

We define a debt write-down as a debt rollover where all payments due in current period are suspended and there is a haircut in the present value of the existing. Formally:

Definition 4. An admissible debt write-down is a debt restructuring proposal (r', I) with $r' < r$, (otherwise, the restructured debt will have negative present value) and $I = \left[(1+r)K - y_1 + \underline{y}\right]$.

The next proposition demonstrates that even when a debt standstill isn't sustainable, there is always an admissible debt write-down that is sustainable:

Proposition 4: *For $I = (1+r)K - y_1 + \underline{y}$, there is an admissible sustainable debt write-down (r', I) with $1 + r' \leq Eg(I) - \varepsilon(I)$; moreover, if $G'(I) > 0$, any admissible sustainable debt write-down (r', I) must have $r' \leq \hat{R}(I)$.*

Proof. That there is an admissible sustainable debt write-down (r', I) with $r' \leq Eg(I) - \varepsilon(I)$, $I = (1+r)K - y_1 + \underline{y}$ is an immediate consequence of Proposition 1. Again, if $G'(I) > 0$,by Proposition 2, any admissible sustainable debt write-down (r', I) must have $r' \leq \hat{R}(I)$. ■

In terms of Figure 3, for $I = (1+r)K - y_1 + \underline{y}$, even if $1 + r > \hat{R}(I)$, we can always find a r' s.t. $1 + r' \leq \hat{R}(I)$ which would then entail a debt write-down.

3.2 Calibration

A recent report by the Economic Commission for Africa (2020) notes that relative to an initial forecast of 3.2%, African growth rate in 2020 is likely to decline to 1.8 % (in the best case scenario), 0.1% (in the middle case scenario) to -2.6% (in the worst case scenario). They also point out that the yield on 10-year government bonds varies from 11.03% for South Africa to 16.67% for Uganda. This has the immediate implication that a straightforward standstill will have little or no effect in ensuring debt sustainability for African countries. The same report suggests that total stock of outstanding debt in Africa is $400 billion and calls for $200 billion Covid 19 related funding for Africa (including low and middle income countries).

Using the data reported in the above report, we carry out a simple calibration to quantify the impact of changes in investment I on the upper bound on the interest rate at which debt can be sustainably restructured. We remind the reader that when the upper bound is lower than one, this corresponds to a scenario where at least some of the debt being restructured takes the form of grants and involves a debt write-down.

In calculating the upper bound on the interest rate at which debt can be sustainably restructured, a key variable is the ratio $\frac{I}{K}$ i.e. the ratio of investment to the initial bond issue. In this calibration exercise, we will assume that $K = \$400$ billion and vary I (and $p(I)$) and calculate the upper bound on the interest rate at which debt can be sustainably restructured to be $1 + r' = \hat{R}(I)$.

We set the no recovery annual growth rate to 0% (close to the middle case scenario of 0%) and 5% (the average growth rate across the African continent over the last decade) over as the annual growth rate in the recovery phase. It is also worth noting that ours is a two-period model where the last period is assumed to represent growth prospects in the medium term. So we when we talk about growth i.e. $EG(I) = \frac{p(I)\left[y_H - \underline{y}\right] + (1-p(I))\left[y_L - \underline{y}\right]}{I}$, we will use compounded growth rates over a 20 year period to calculate $\frac{\left[y_H - \underline{y}\right]}{I}$ and $\frac{\left[y_L - \underline{y}\right]}{I}$ respectively (see Appendix 1 for a formal justification).

Consider, first, a scenario where $I = K = \$400$ billion. Assuming that in this case, $p(I) = 0.99$, we obtain that upper bound on the interest rate at which debt can be sustainably restructured is $\hat{R}(I) = 1.2424$. If, instead, we assume that $I = \$300$ billion and $p(I) = 0.95$ then under the same assumptions on expected growth we obtain that the upper bound on the interest rate at which debt can be sustainably restructured to be $\hat{R}(I) = 1.0829$. Next, we assume that $I = \$200$ billion and $p(I) = 0.85$ then we obtain that the upper bound on the interest rate at which debt can be sustainably restructured to be $\hat{R}(I) = 0.8664$. Of course, if we assume that $I = \$100$ billion and $p(I) = 0.65$, then the upper bound on investment is on the interest rate at which debt can be sustainably restructured to be $\hat{R}(I) = 0.5706$. The following table summarizes the calibration:

	Upper bound on interest rate $\hat{R}(I)$
$I = \$400\ billion, p(I) = 0.99$	1.2424
$I = \$300\ billion, p(I) = 0.95$	1.0829
$I = \$200\ billion, p(I) = 0.8$	0.8664
$I = \$100\ billion, p(I) = 0.99$	0.5706

Table: Calibration Results

So, the level of investment (including outstanding debt payments rolled over and fresh investment) has to be high enough to ensure that there is a positive interest rate at which sustainable debt restructuring can take place. However, as the ratio of investment to the existing stock of bond issue becomes lower, there is no positive interest rate at which debt restructuring can take place and in practice, must include a mix of debt write down and additional finance in the form of outright grants and loans at very low interest rates.

In a recent report, the IMF (IMF 2020) points out that countries across Africa still face financing needs amounting to over $110 billion in 2020, with $44 billion still having to be financed; this estimate is lower than the numbers estimated by the United Nations Economic Commission for Africa. However, given our preceding analysis, whatever be the estimate of the financing needs, the key to restoring sustainability will be the intertest rate used. Our analysis suggests that initiatives such as the Rapid Credit Facility (RCF) and the Rapid Finance Initiative (RFI) which provides rapid concessional financial assistance with limited conditionality to low-income countries (LICs) facing an urgent balance of payments need with an emphasis on the country's poverty reduction and growth objectives. Disbursements under RCF are at zero interest rates and our analysis suggests that such funding needs to be expanded by an order of magnitude. What is key though is that the money accessed under such an initiative is used by the country for its poverty reduction, public health and growth needs and not to bail out private creditors.

3.3 Creditor Heterogeneity and Participation

A key issue with the kind of debt restructuring proposals we have discussed is the willingness of all creditors to participate in such a process. Although for the purposes of the formal analysis we have assumed that creditors, whether official or private, act collectively as a group, in practice this unlikely to be the case. Disbursements made by one group of creditors are used, by the debtor state, to make payments to another group of creditors. For instance, the Jubilee Debt Campaign have pointed out that 28 countries at high risk of debt default had received $11.3bn (£8.9bn) that would be used to meet private sector debt commitments (see https://www.theguardian.com/business/2020/jul/16/dollar-11bn-in-imf-covid-19-money-being-used-to-service-debt-says-group-private-lendwers-poorest-nations).

When a portion of disbursements is used to meet existing private sector commitments, then analysis of debt sustainability needs to be modified. As before, given (r', I), the present value of the refinanced debt will, as before, be $(1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K$. If $z \in [0, (1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K]$ is used to payoff existing creditors, then in expected terms, the total resources available is $p(I-z')\left[y_H - \underline{y}\right] + (1 - p(I-z'))\left[y_L - \underline{y}\right]$ where $z' \leq z$ (to allow for the possibility that private sector investment may be crowded in by debtor state when it meets its private sector debt commitments). Hence, for restructuring to be sustainable (in expected terms), we must have

$$(1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K \leq p(I-z')\left[y_H - \underline{y}\right] + (1 - p(I-z'))\left[y_L - \underline{y}\right]$$

so that expected growth is given by the expression:

$$EG(I-z) = \frac{p(I-z')\left[y_H - \underline{y}\right] + (1 - p(I-z'))\left[y_L - \underline{y}\right]}{(1+r)K - y_1 + \underline{y} + X} = \frac{E\pi(I-z')}{I}$$

Of course, unless $z' = 0$, there conditions for sustainability are now going to be more restrictive:

Proposition 4: *A necessary and sufficient condition for an admissible debt restructuring proposal* (r', I) *when* $z \in [0, (1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K]$ *is used to payoff existing creditors and an amount* $z - z' \geq 0$ *is private sector investment is crowded in is* $1 + r' \leq 1 + Eg(I-z') - \tilde{\varepsilon}(I, z')$ *where* $0 < \tilde{\varepsilon}(I, z') < EG(I-z')$.

Proof. The proof essentially follows that of Proposition 1. Let $\varepsilon(I, z') = EG(I-z') - x$. Then, we can compute the admissible value of $\varepsilon(I, z')$ to be

$$\varepsilon(I,z') = \frac{\left(2EG(I-z') + \frac{I}{K}\right) - \sqrt{\left(\frac{I}{K}\right)^2 + \frac{4EG(I-z')I}{K}}}{2}$$

The conclusion follows. ■

As the United Nations Economic Commission for Africa notes private sector debt makes up a disproportionate share of the debt-servicing cost. For several African countries, even where debt levels remain relatively low, the interest cost now accounts for 20 percent or more of government revenues. Without significant private sector participation, the standstill will fall short of its objectives.

We can use the calibration exercise in Section 3.2. to illustrate just how much restrictive matters can become. Consider the scenario where that $I = \$100$ billion and $z = \$22$ billion (half of the $\$44$ billion in debt repayments due). Assuming $z' = \$20$ billion (so that $I - z' = \$80$ billion) and $p(I - z') = 0.55$, we obtain that the upper bound on the interest rate at which debt can be sustainably restructured to be $\hat{R}(I) = 0.5186$. Of course, if we assume that $I = \$100$ billion and $p(I) = 0.65$ (when $z = 0$), then the upper bound on investment is on the interest rate at which debt can be sustainably restructured to be $\hat{R}(I) = 0.5706 > 0.5186.$

4. Investment and the Political Economy of Debtor Moral Hazard

The key implication of the analysis carried out so far is the need for investment (including fresh financing) to ensure debt sustainability in the face of a severe negative shock like Covid 19. The assumption made so far is that all the investment made in the debtor state will be used for the purpose it is intended for i.e. maximize productive investments to increase the prospect of recovery. This assumption is moot.

In this section, we consider the issue of debtor moral hazard from the perspective of its domestic political economy. Domestic agents are split into two groups, a minority elite and a majority non-elite. The domestic elite participate in international capital markets and are organised to engage in collective political activity ((Olson (1965)). Taken together, these two assumptions imply that the resources appropriated by the elite cannot be used for debt repayments without their explicit consent. The non-elite cannot directly participate in international capital markets and, initially, are not organised to engage in collective political activity. Their payoffs are derived from solely from domestic national income.

Conditional on a negative shock, we characterize the different incentives of the domestic elite and non-elite to deploy the investment made as part of the debt restructuring and show that effective participation by the domestic non-elite in the debt restructuring process will lead to sustainability and economic recovery.

4.1. Domestic Elite Incentives

The population in the debtor state is normalised to one. Domestic non-elites constitute a majority and the domestic elites a minority. For ease of exposition, we will assume both elites and non-elites have risk neutral preferences over consumption in period 2.

Suppose the creditors agree to a non-contingent debt proposal (r', I) where only elites participate in the debt restructuring process and have the power to divert x from (from I) for their own benefit and moreover, such a decision is non-contractible as part of the debt restructuring process. This money is invested in a foreign asset with a return of r^f (where r^f could be equal to the lower bound $\underline{r} > 0$ on the interest rate at which debt can be restructured given by creditor participation constraints). In addition, the elites a fraction β of any domestic output that is left over debt repayment has been made. Therefore, their payoffs from diverting resources x from public investment is:

$$g(x) = x(1 + r^f) + p(I - x)\beta w_H + (1 - p(I - x))\beta w_L$$

where $w_H = y_H - (1 + r')\left[(1 + r)K - y_1 + \underline{y} + X\right] + (1 + r')^2 K$ and $w_L = min\left\{\underline{y}, y_L - (1 + r')\left[(1 + r)K - y_1 + \underline{y} + X\right] + (1 + r')^2 K\right\}$.

The following proposition characterizes the decision made by the elite:

Proposition 5: *Given a debt restructuring proposal* (r', I), *then the elite's optimal choice to divert resources* x^* *is characterized by the following: (i) if* $(1 + r^f) \leq p'(I)\beta(w_H - w_L)$, *then* $x^* = 0$; *(ii) if* $p'(0)\beta(w_H - w_L) \leq (1 + r^f)$, $x^* = I$; *(iii) if* $p'(0)\beta(w_H - w_L) > (1 + r^f) > p'(I)\beta(w_H - w_L)$, $0 < x^* < I$, $\frac{dx^*}{dr^f} > 0$, $\frac{dx^*}{dI} = 1$ *and* x^* *is non-decreasing in* r'.

Proof. By computation:

$$g'(x) = (1 + r^f) - p'(I - x)\beta[w_H - w_L], f''(x) = p''(I - x)\beta[w_H - w_L]$$

As $p''(I) < 0$, $g''(x) < 0$ so that the maximization problem is concave. Hence, if $(1 + r^f) \leq p'(I)\beta(w_H - w_L)$, then $x^* = 0$; if $p'(0)\beta(w_H - w_L) \leq (1 + r^f)$, $x^* = I$; if $p'(0)\beta(w_H - w_L) > (1 + r^f) > p'(I)\beta(w_H - w_L)$, then $0 < x^* < I$. When $0 < x^* < I$, the FOC is

$$(1 + r^f) = p'(I - x^*)\beta[w_H - w_L]$$

Let $w = w_H - w_L$. Then, taking the total derivate of the FOC w.r.t. r^f, I, x^*, w we obtain that

$$dr^f = -p''(I - x^*)\beta[w_H - w_L]dx^* + p''(I - x^*)\beta[w_H - w_L]dI + p'(I - x^*)\beta dw$$

so that

$$\frac{dx^*}{dr^f} = \frac{1}{-p''(I - x^*)\beta[w_H - w_L]} > 0, \frac{dx^*}{dI} = \frac{p''(I - x^*)\beta[w_H - w_L]}{p''(I - x^*)\beta[w_H - w_L]} = 1$$

Now, $\frac{dx^*}{dw} = \frac{p''(I-x^*)\beta[w_H - w_L]}{p'(I-x^*)\beta} > 0$. Also, note that if $\underline{y} \le y_L - (1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K$, then $w = w_H - w_L = y_H - y_L$; otherwise, $w = w_H - w_L = y_H - (1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K - \underline{y}$. Hence, x^* is weakly decreasing in r' as required. ■

Given Proposition 5, we need to modify the definition of sustainability. Given (r', I), the present value of the refinanced debt will, as before, be $(1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K$. In expected terms, given the elite decision to divert resources x^*, the total resources available is $p(I-x^*)\left[y_H - \underline{y}\right] + (1 - p(I-x^*))\left[y_L - \underline{y}\right]$. Hence, for restructuring to be sustainable (in expected terms), we must have

$$(1+r')\left[(1+r)K - y_1 + \underline{y} + X\right] + (1+r')^2K \le p(I-x^*)\left[y_H - \underline{y}\right] + (1 - p(I-x^*))\left[y_L - \underline{y}\right]$$

so that expected growth is given by the expression:

$$EG(I-x^*) = \frac{p(I-x^*)\left[y_H - \underline{y}\right] + \left(1 - p(I-x^*)\right)\left[y_L - \underline{y}\right]}{(1+r)K - y_1 + \underline{y} + X} = \frac{E\pi(I-x^*)}{I}$$

Proposition 1 needs to be restated as follows (the proof follows exactly the same steps as Proposition 4 and is omitted):

Proposition 4’: *A necessary condition for an admissible debt restructuring proposal* (r', I)*, given the elite’s optimal choice to divert resources* x^* *to be sustainable is* $r' \le Eg(I-x^*)$ *; the sufficient condition is* $1 + r' \le 1 + Eg(I-x^*) - \varepsilon(I, x^*)$ *where* $0 < \varepsilon(I, x^*) < EG(I-x^*)$*.*

Evidently, Proposition 4’ is more restrictive than Proposition 1: when elites have the power to divert resources, then this restricts the set of sustainable debt restructurings.

4.2 Political Economy of Debt Restructuring

Non-elites cannot access international capital markets with the immediate implication that even if they have the power to divert resources, they will choose not to do so as their payoffs are tied to the performance of the domestic economy and hence, the returns from investment.

What factors determine whether the elite or the non-elite have the decision-making power to restructure debt? As the elite are organised (i.e. are able to act collectively in their own interests) and the non-elite aren’t, they will be the decision-makers. For the non-elite to acquire decision-making power, they must be able to organise to act collectively.

Following Olson (1965), assume that an organisation successfully engages in collective political activity (such as a political party or a labour union) whose members are rewarded selectively. Each individual in the non-elite has the choice of becoming a party member; joining the party is costly and becomes a dominant strategy for an individual if and only if the number of other individuals joining the party is greater than a critical threshold. This suggests that there will be two possible outcomes in equilibrium, one where the non-elite are organised along party or union lines and are able to act collectively to grab the decision-making power over debt restructuring and one in which they remain disorganised and the decision-making power over debt restructuring agents remains vested in the elite. Which outcome prevails will depend on how non-elite individuals solve the underlying coordination problem.

Let $\Delta= [p(I) - p(I - x^*)](1 - \beta)[w_H - w_L] > 0$ denote the net payoff gain to the non-elite (as a group) when investment is deployed to ensure economic recovery relative to the situation when it isn't. Let π denote the fraction of the non-elite who join the party. It will be assumed that the probability with which the non-elite get to decide whether or not debt is restructured is given by a function $f(\pi)$, strictly increasing in π, with $f(0) = 0$ and $f(1) \leq 1$. Given $\pi \in [0,1]$, the net payoff gain to a non-elite party member is $f(\pi)\Delta - c$ where c is the cost of joining the party while the net payoff gain to a non-elite individual who is not a party member is $\gamma f(\pi)\Delta$ where $\gamma \geq 0$ is a small non-negative number close to zero, strictly less than 1.

Clearly if $c \geq f(\pi)\Delta$, then it is a dominant action (strictly dominant when the inequality is strict) for no non-elite individual to join the party and engage in collective action.

Suppose $0 \leq \gamma f(\pi) < c < f(\pi)\Delta$. Then, there exists a function $\tilde{\pi}(c), 0 < \tilde{\pi}(c) < 1$ such that that it is a dominant action for each non-elite individual to join the party if and only if $\pi > \tilde{\pi}(c)$, where $\tilde{\pi}(c)$ is the unique solution to the equation $f(\pi)\Delta - c = \gamma f(\pi)$. As $f(\pi)$ is strictly increasing in π, $\tilde{\pi}(c)$ is strictly decreasing as a function of c.

(i) When $c > f(1)\Delta$, it is a dominant action for each non-elite individual not to join the party. Hence, only the elite have the decision-making power over debt restructuring.

(ii) When $f(1)\Delta >$ c, it is a dominant action for each non-elite individual to join the party. Hence, the non-elite have the decision-making power over debt restructuring with probability $\mathrm{f}_{\max}$.

(iii) When $0 \leq \gamma\mathrm{f}(1)\Delta < c < \mathrm{f}(1)\Delta$, (a) if $\pi = 0$, it is best response for each non-elite not to join the party, and (b) if $\pi = 1$ it is a best response for each non-elite individual not to join the party. Hence, there are two Nash equilibria, one where no non-elite individual joins the party and one where all non-elite individuals joins the party. It remains to be seen which of the two equilibria will be selected?

We will use the notion of a stochastically stable equilibrium (Young (1998)) to select between the two equilibria in the coordination game played by the non-elite. Let G be an arbitrary finite normal form game with a set of N players, an action set A^i for each player $i = 1,..,N$ and a payoff $u^i: \prod_{i=1}^{N} A^i \to \mathfrak{R}$. Suppose each player believes that whenever any other player chooses to play a specific action, with probability $\theta, 0 < \theta < 1$, she ends up choosing some other action in A^i . Let $G(\theta)$ denote the perturbed game. A state in $G(\theta)$ is a profile of actions. For each state, let each player pick a best response to that state in $G(\theta)$ i.e. taking into account the possibility that other individuals will make a mistake with probability θ. This defines a function σ from the set of states to itself. If there are many best responses, then there will be many such functions σ. When θ is small enough, let the set of $\sigma's$ that remain best responses for all smaller θ be denoted by $S(G)$. Any $\sigma \in$

$S(G)$. together with θ defines a Markov process over the set of states that is both irreducible and aperiodic and therefore has a unique steady-state distribution. A stochastically stable state is one which has positive probability under the limit of the steady state distribution of the preceding Markov process as θ goes to zero for any selection $\sigma \in S(G)$. If a state is both a Nash equilibrium of G and a stochastically stable, then it is said to be a stochastically stable equilibrium of G.

The following proposition characterizes which equilibrium will be selected:

Proposition 6. *Suppose* $0 \leq \gamma f_{max}\Delta < c < f_{max}\Delta$, *then there are two equilibria, one with positive probability f(1) of party formation with full non-elite participation in the debt restructuring process and another with no party formation and no non-elite participation in debt restructuring process. The equilibrium where all non-elite individuals join the party is when* $\tilde{\pi}(c) < \frac{1}{2}$.

Proof. As there is a continuum of non-elite individuals while the definition of stochastic stability presupposes a game with a finite number of players, we proceed as follows. Consider a sequence of finite grids contained in the mass of the non-elite individuals whose limit is the mass of non-elite individuals. Denote such a sequence of finite grids by $\widehat{N_J}, j \geq 1$. Let $N_j = \#\widehat{N_J}$. We call a sequence of finite grids admissible if (i) there is a threshold $\widetilde{N_J}$, for each $j \geq 1$ such that $lim_{j\to\infty} \frac{\widetilde{N_J}}{N_j} = \tilde{\pi}(c)$, (ii) the payoff to a party member is $f(\pi)\Delta - c$ if the number party members is greater than or equal to $\widetilde{N_J}$ and is $-c$ otherwise, (iii) the payoff to a non-party member is $\gamma f(\pi)\Delta$. We say that an equilibrium to be stochastically stable in the coordination game played by the non-elite, it must be the limit of the sequence of stochastically stable equilibria of all admissible sequences of finite grids converging to the mass of the non-elite. Fix j and consider $\widehat{N_J}$. For θ small enough, if at least $\widetilde{N_J}$ non-elite individuals join the party, then the best response of each non-party member of the non-elite must be to choose join the party as well. Similarly, if at most $\widetilde{N_J} - 1$ join the party, then the best response of each non-party member must be not to join the party. In states where exactly $\widetilde{N_J} - 1$ join the party, choosing either of the two options, join the party or not join the party, are possible best responses for an individual belonging to the non-elite. It follows that that best responses differ only in states where the number of individuals choosing to join the party is exactly $\widetilde{N_J} - 1$. Now, consider the associated Markov process for small θ. There are two recurrent communication classes (for the definition of the terms "recurrent communication classes", "resistance" and "minimum stochastic potential", see Young (1998)), one where all non-elite individuals choose to join the party (labelled a) and one in which all non-elite individuals choose not to join the party (labelled b). By Theorem 4 in Young (1993), only states in a recurrent communication class with least resistance will have positive probability weight in the limit of the steady state distribution of the Markov process as θ goes to zero. Consider the state b. Then, (i) there is a best response selection such that given $N_j - \widetilde{N_J} + 2$ errors, the best response of each individual is to be in a and (ii) there is a best response selection such that given $N_j - \widetilde{N_J} + 1$ errors, the best response of each individual is to be in a. Therefore, the minimum resistance of leaving the state b, depending on the selection made, is either $N_j - \widetilde{N_J} + 1$ or $N_j - \widetilde{N_J} + 2$. It follows that the minimum resistance of a tree oriented from the state b to the state a, depending on the best response selection made, is either $N_j - \widetilde{N_J} + 1$ or $N_j - \widetilde{N_J} + 2$. Next, consider the state a. Then, there is both a best response selection such that given $\widetilde{N_J} - 1$ errors, the best response of each individual is to be in **b**, and a best response selection such that given $\widetilde{N_J} - 2$ errors, the best response of each individual is to be in **b**. Therefore, the minimum resistance of leaving the state **a**, depending on the best response selection is either $\widetilde{N_J} - 1$ or $\widetilde{N_J} - 2$. It follows that the minimum resistance of a tree oriented from the state **a** to the state **b**, depending on the best response selection made, is also either $\widetilde{N_J} - 1$ or $\widetilde{N_J} - 2$. The state **b** is the unique stochastically stable equilibrium if and only if both $N_j - \widetilde{N_J} + 1 < \widetilde{N_j} - 1$ and $N_j - \widetilde{N_J} + 2 < \widetilde{N_J} - 2$ or equivalently, both $\widetilde{N_J} > \frac{N_j+2}{2}$ and $\widetilde{N_J} > \frac{N_j+4}{2}$. As $\frac{N_j+2}{2} > \frac{N_j+4}{2}$ if $\widetilde{N_J} - 2 > \frac{N_j}{2}$, the state **a** is the unique stochastically stable equilibrium. Rewriting these inequalities, it follows that state **a** is the unique stochastically stable equilibrium if and only if $\frac{\widetilde{N_J}-2}{N_j} > \frac{1}{2}$. For any admissible sequence of finite grids, $lim_{j\to\infty} \frac{\widetilde{N_J}-2}{N_j} = \tilde{\pi}(c)$ so that when $\tilde{\pi}(c) > \frac{1}{2}$, the unique stochastically stable equilibrium is one where all non-elite individuals do not join the party or conversely, when $\tilde{\pi}(c) < \frac{1}{2}$, the unique stochastically stable equilibrium is one where all non-elite individuals join the party. ■

Proposition 6 sets out the conditions under which the non-elite, by organising themselves along party lines, engage in collective action to obtain decision-making power over the decision to restructure debt. First, the probability of successfully usurping decision-making power, conditional on being organised along party lines, is above a certain threshold ($f(1) > \frac{c}{\Delta}$). If, to the contrary, $f(1) < \frac{c}{\Delta}$ is very low, then even when the non-elite are fully organised along party lines and able to engage in collective action cannot win the decision-making power to restructure debt. Anticipating such an outcome, no non-elite individual will decide to engage in collective in first place and the elite will retain decision-making power. Second, the cost paid each individual in the non-elite to engage in collective action is below a certain threshold (i.e. $c < \tilde{\pi}^{-1}(\frac{1}{2})$). Note that c, the cost to each non-elite individual of engaging in collective political activity, is a measure of how democratic a country is. In particular, in a dictatorship, c will be high while in a democracy c will be lower in value. For moderate levels of c, in Proposition 6, it is shown that each non-elite individual's expectations on other non-elite individuals' most likely course of action is that they will choose to participate; such a belief, when there are multiple equilibrium outcomes, acts as an equilibrium coordination device, inducing the non-elite individuals to their collective action problem. When the c is very low in value it becomes a dominant action for each non-elite individual to participate in collective action.

4.3. Reviving the UNCTAD Road Map for Africa

The suggestion that the non-elite citizens affected by a sovereign debt crisis have claims that are justifiable and independent of the elite contests the assumption that a sovereign debtor should be thought of as a representative agent. In the time of the Covid 19 pandemic. Domestic elite has its own economic interests in a debt crisis which are distinct from domestic non-elites whose interests are linked to sustainable debt restructuring.

Our formal analysis implies that the participation in the debt restructuring process by citizens of the debtor state is key to restoring sustainability. This raises issues about the design of institutions that enhance the role of non-elite citizen groups in a debt restructuring. This is a point explicitly raised in the UNCTAD road map (UNCTAD 2015) that explicitly accounts for such a role at several points in the lead up to a debt restructuring.

The motivation behind the UNCTAD roadmap was the ‘socialisation of losses from private debts and the subsequent emergence of sovereign debt crisis in developing and developed countries.’ (UNCTAD, pp. 3) The proposal aims to enhance ‘coherence, fairness and efficiency of sovereign debt workouts.’ (Id.). The proposals set out ‘specific recommendations for each step of a sovereign debt workout.

A key aspect of each recommendation is the explicit recognition and acknowledgement of civil society as an independent constituency whose interests are both distinct from those of the debtor government and the formal creditors. For instance, the principle of impartiality recognises that debt workouts need to be defined by a ‘neutral perspective particularly with regard to sustainability assessments and decisions about restructuring terms’ (UNCTAD, pp. 4) rather than as a procedure to fulfil the self-interest of either the debtors or the creditors. Further, the issue of ‘sustainability requires that sovereign debt workouts are completed in a timely and efficient manner...while minimizing costs for economic and social rights and development in the debtor state.’ (Id.) Debt restructuring must restore debt sustainability which would then limit the problem.

There are two ways in which an UNCTAD road map is reflects our analysis. First there is a recognition that there is a common interest between the debtor state, controlled by an elite with decision-making power over restructuring debt and international capital markets. Second, this interest is not shared by the domestic non-elite: hence the need for independent intervention by citizens of a debtor state.

This leads us to suggest a revival of the UNCTAD road map for Africa as a key component of debt restructuring in response to Covid 19. As UNECA (2020) notes, several African countries have come under the spotlight for governance-related matters: 24 African countries had Country Policy and Institutional Assessment (CPIA) scores in the public sector management that below the African average of 3.1 (out of a maximum of 6).

Women, in particular, have been disadvantaged due to rising domestic violence, “the closure of schools and the diminished protection from governments create an enabling environment for child marriage and sex transactions between young girls and older men as a means of economic survival for families.” (UNECA 2020). Furthermore, 65% of nurses are women who are key workers at the frontline of delivery of health care services and they carry out the bulk of unpaid work in several countries (UNECA (2020)). The same report notes the need for a $15 billion fund for resourcing health care, targeted public health campaigns, protecting health workers, procurement of medical equipment and personal protective equipment through WHO and CDC.

All of this suggests that women, public health practitioners, key workers, community groups, civil society organisations participate in the debt restructuring process to determine the financing needs of the debtor state, the interest rate at which the financing occurs as well as the use of the finance within the debtor state.

However, our analysis results suggest that for the UNCTAD roadmap to work in practice, the institutions it creates must increase f_{max} and lower c. Only then will the domestic non-elite collectively organise to grab the opportunity to obtain decision-making power over debt restructuring.

5. Conclusion

Given the magnitude of the global negative shock resulting from the Covid 19 pandemic, how official and private external creditors respond to restructure debt for low income sovereign states is likely to have long lasting impacts on their prospects for recovery. In this paper, we have taken a first step towards providing a formal analysis of this problem. Our broad conclusions are as follows: (a) sustainable debt restructuring must involve a mix of debt write down and financing in the form of outright grants and loans at very low interest rates, and (b) participation in the debt restructuring process by community groups, civil society organisations is key to restoring sustainability.

In future work, we plan to extend our formal analysis to more general settings with a focus on quantification and empirical analysis.

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Appendix 1: Deriving $\boldsymbol{p(I)}$ and $\boldsymbol{G(I)}$ from an endogenous growth model

Consider the following simple AK endogenous growth adapted from Sachs et. al. (2004). Let k denote the capital labour ratio and $A(k)$ denote a technology parameter affecting total factor productivity. Per capita output y is described the following equation:

$$y = A(k)k$$

and capital accumulation by the following equation:

$$\frac{dk}{dt} = s(k)A(k)k - (n+d)k$$

where $s(k)$ denotes the per capita savings rate denotes, n the population growth rate, d the depreciation rate. The growth rate of per capita output id given by the equation:

$$\frac{1}{k}\frac{dk}{dt} = s(k)A(k) - (n+d)$$

Suppose there is a threshold level of per capita capital $\hat{k}$ such that $s(k)A(k) \leq (n+d)$ whenever $k < \hat{k}$ and $s(k)A(k) > (n+d)$ when $k > \hat{k}$.

Suppose $\hat{k}$ is distributed in some interval $[\underline{k}, \overline{k}]$ according to the distribution $F(\hat{k})$. If the initial level of per capita capital is k_0 and I is the level of investment, then we set $p(I) = F(k_0 + I > \hat{k})$.

The immediate implication is that when k_0, the initial level of per capita capital is lower than $\hat{k}$, then output per capital converges to zero over time: taken cumulatively over a T year horizon (where $T = 15$ or $T = 20$), this corresponds to the low output growth scenario $\pi_L - \underline{\pi}$ studied in the main text). When $k_0 > \hat{k}$ output per capita keeps growing over time: taken cumulatively over a T year horizon (e.g. $T = 20$), this corresponds to the high growth scenario $\pi_H - \underline{\pi}$ studied in the main text. This interpretation is used explicitly in the calibration.

Appendix 2: Contingent Debt

In our analysis so far we have made the assumption that the interest rate at which debt is not contingent on a future state of the world. This has the consequence that the sustainability constraint holds as in expected terms so that there could be a future state of the world where the restructured debt may not be repaid. To ensure that sustainability holds in all future states of the world, we need to consider contingent debt restructuring where the interest rate paid on sovereign debt is contingent on the state of the world that prevails. For example, the interest rate could be linked to future GDP or future export earnings. This leads us to the following question: can a contingent debt contract restore sustainability when a non-contingent debt contract cannot?

Formally, we define a contingent debt restructuring proposal as follows:

Definition. The triple (r_H', r_L', I) is a contingent debt restructuring proposal. It is admissible if $1 + r_H' > 0$ and $1 + r_L' > 0$.

For clarity of exposition, in this appendix, we will refer a debt structuring proposal (r', I) as a non-contingent debt restructuring proposal.

Consider what happens if creditors (as a group) choose to advance $(1+r)K - y_1 + \underline{y}$ to the sovereign via a contingent debt restructuring proposal (r'_H, r'_L, I). Debt sustainability requires that the following two inequalities simultaneously hold:

$$(1+r'_H)\left[(1+r)K - y_1 + \underline{y} + I\right] + (1+r'_H)^2 K \leq \left[y_H - \underline{y}\right]$$

$$(1+r'_L)\left[(1+r)K - y_1 + \underline{y} + I\right] + (1+r'_L)^2 K \leq \left[y_L - \underline{y}\right]$$

The contingent return to the amount invested is

$$G_H(I) = \frac{\left[y_H - \underline{y}\right]}{(1+r)K - y_1 + \underline{y} + I} = \frac{\pi_H}{I}, G_L(I) = \frac{\left[y_L - \underline{y}\right]}{(1+r)K - y_1 + \underline{y} + I} = \frac{\pi_L}{I}$$

and the contingent rate of growth is $g_H(I) = G_H(I) - 1$ and $g_L(I) = G_L(I) - 1$. Note that $p(I)G_H(I) + \left(1 - p(I)\right)G_L(I) = EG(I)$ and $p(I)g_H(I) + (1 - p(I))g_L(I) = Eg(I)$.

Given a contingent debt restructuring proposal (r'_H, r'_L, I), let $r' = p(I)r'_H + \left(1 - p(I)\right)r'_L$. A contingent debt restructuring proposal (r'_H, r'_L, I), has the same (expected) present value as a non-contingent debt restructuring proposal (r', I) when $r' = p(I)r'_H + \left(1 - p(I)\right)r'_L$.

The following proposition provides a basic characterization for a sustainable contingent debt restructuring proposal:

Proposition: (a) *A necessary condition for a triple is a contingent debt restructuring proposal (r'_H, r'_L, I) to be sustainable is $1 + r'_L \leq 1 + g_L(I)$ and $1 + r'_H \leq 1 + g_H(I)$; the sufficient condition is that $1 + r'_L \leq 1 + g_L(I) - \varepsilon_L(I), 0 < \varepsilon_L(I) < 1 + g_L(I)$ and $1 + r'_H \leq 1 + g_H(I) - \varepsilon_H(I), 0 < \varepsilon_H(I) < 1 + g_H(I)$. (b) Moreover, any sustainable contingent debt restructuring proposal (r'_H, r'_L, I) must have a sustainable non-contingent debt proposal (r', I) with the same expected present value but the reverse may not be hold i.e. for $1 + r' = 1 + p(I)r'_H + \left(1 - p(I)\right)r'_L \leq 1 + Eg(I) - [p(I)\varepsilon_H(I) + \left(1 - p(I)\right)\varepsilon_L(I) < 1 + Eg(I) - \varepsilon(I)$.*

Proof. Then proof of Part (a) follows the steps as the Proof of Proposition 1. Let $x_H = (1 + r'_H)$ (respectively, $x_L = (1{+}r'_L)$. Note that we can rewrite the inequalities characterizing sustainability as

$$x_H + x_H^2 \frac{K}{I} \leq G_H(I), x_L + x_L^2 \frac{K}{I} \leq G_L(I)$$

As both $x_H^2 \frac{K}{I} \geq 0$ and $x_L^2 \frac{K}{I} \geq 0$, we have that $x_H = (1 + r'_H) \leq G_H(I) = 1 + g_H(I)$ and $x_L = (1 + r'_L) \leq G_L(I) = 1 + g_L(I)$. Next, let $\varepsilon_H(I) = G_H(I) - x_H$ (respectively, $\varepsilon_L(I) = G_L(I) - x_L$. Instead of computing the roots of the equation $x_H + x_H^2 \frac{K}{I} - G_H(I) = 0$ (respectively, $x_L + x_L^2 \frac{K}{I} - G_L(I) = 0$), we compute the roots of the equation $\left(G_H(I) - \varepsilon_H(I)\right) + \left(G_H(I) - \varepsilon_H(I)\right)^2 \frac{K}{I} - G_H(I) = 0$ (respectively, $\left(G_L(I) - \varepsilon_L(I)\right) + \left(G_L(I) - \varepsilon_L(I)\right)^2 \frac{K}{I} - G_L(I) = 0$). The steps in the computation follow the same steps as in Proposition 1 for computing the roots of (2). To avoid needless repetition, we omit these steps and note that the admissible root where $G_H(I) - \varepsilon_H(I) > 0$

(respectively, $G_L(I)-\varepsilon_L(I)>0$) is given the expression $\varepsilon_H(I)=\frac{\left(2G_H(I)+\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4G_H(I)I}{K}}}{2}$ (respectively, $\varepsilon_L(I)=\frac{\left(2G_L(I)+\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4G_L(I)I}{K}}}{2}$. Hence, taking this root, we have that the present value of the restructured debt is non-negative in both states of the world and as the LHS of both (5a) and (5b) is increasing in r'_H, r'_L respectively, the conclusion follows. The proof part (b) is as follows. Note that

$$p(I)[G_H(I)-\varepsilon_H(I)]+\big(1-p(I)\big)[G_L(I)-\varepsilon_L(I)]=EG(I)-\big[p(I)\varepsilon_H(I)+\big(1-p(I)\big)\varepsilon_L(I)\big]$$

Now:

$$p(I)\varepsilon_H(I)+\big(1-p(I)\big)\varepsilon_L(I)=\frac{\left(2EG(I)+\frac{I}{K}\right)-[p(I)\sqrt{\left(\frac{I}{K}\right)^2+\frac{4G_H(I)I}{K}}+(1-p(I)\sqrt{\left(\frac{I}{K}\right)^2+\frac{4G_L(I)I}{K}})}{2}$$

$$>\frac{\left(2EG(I)+\frac{I}{K}\right)-\sqrt{\left(\frac{I}{K}\right)^2+\frac{4EG(I)I}{K}}}{2}=\varepsilon(I)$$

where the penultimate inequality follows from Jensen's inequality for concave functions because $\sqrt{\left(\frac{I}{K}\right)^2+\frac{4yI}{K}}$ is concave in y. Therefore, $EG(I)-[p(I)\varepsilon_H(I)+\big(1-p(I)\big)\varepsilon_L(I)<EG(I)-\varepsilon(I)$ as required. ■

The above proposition shows that any sustainable contingent debt restructuring proposal (r'_H, r'_L, I) must have a sustainable non-contingent debt proposal (r', I) with the same expected present value but the reverse may not be hold. The intuition is that the upper bound on the interest rate calculated in Propositions and 1 and 2 in the main text are concave in the expected growth rate. A key implication is that there cannot be a sustainable contingent debt restructuring proposal with the same expected present value as an unsustainable non-contingent debt restructuring proposal. Hence, as in the main text, faced with an unsustainable non-contingent debt restructuring proposal, a sustainable contingent debt proposal must entail a debt write-down is expected terms or additional financing or both.